AVAILABLE PHD PROJECTS

MILANO LOGIC GROUP

The Logic Group at the Department of Philosophy, University of Milan, plays an active in the PhD programme *Mind, Brain and Reasoning* for which four full-time positions are now open. The call with full application details is available on page 25 of this document. **The strict deadline for applications is 28 June 2021**.

The group is willing to supervise outstanding candidates on either of the two projects described below. Prospective candidates are welcome to email us with informal enquiries.

1. A conditional perspective on Probability Logic

1.1. **Background.** Reasoning is at root conditional, and so is every logical model of it. This is why the notion of logical consequence plays such a central role in the foundations and applications of logical systems. Classical logic provides a well understood setting of a notion of conditional reasoning whose scope is limited essentially to mathematical reasoning. A key question of logical, scientific and philosophical interest arises as to how to put to use – in a methodologically principled way – the logico-mathematical virtues of classical logic in more general contexts of reasoning. For the wide domain of reasoning under uncertainty, this question was a key motivation for the seminal contributions of George Boole and Augustus De Morgan who, in the 1850s, realised that the then-incipient mathematical theory of probability could be coupled with logical reasoning, provided this could be tackled algebraically. This resulted in what we now call boolean algebras.

The grandiose plan of capturing jointly the "The Calculus of Inference, Necessary and Probable", as stated in the subtitle of De Morgan's "Formal Logic", turned out to be essentially a false start. The incipient characterisation of logic as a theory of mathematical inference championed by Frege, Russell and Hilbert on the one hand, and the assimilation of probability to the theories of measure and integration championed by Borel, Lebesgue and

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Kolmogorov on the other hand, resulted in a clear-cut distinction between the two fields, which developed following largely non-intersecting paths.

While not mainstream, the idea of combining logic and probability was never abandoned [20–22], especially owing to its relevance to artificial intelligence [16, 32, 35, 36]. The current developments of which, especially in connection to the opportunities and limitations of machine learning are putting again the Boole-De Morgan view on center stage [10, 41].

1.2. **Aim of the project.** The overarching goal of this PhD thesis is to provide

(1) a thorough review of the relevant literature on the logico-probabilistic analysis of conditional reasoning spanning the various relevant approaches, including the philosophical [1, 28, 29], probabilistic [8, 33] and computer science based [5, 7, 13].
(2) an original contribution along *one* of the following lines
   - the application of the framework of Boolean Algebras of Conditionals [14] to the analysis of conditional independence (e.g [19])
   - the conditional-probabilistic analysis of the key patterns in scientific inference, especially *modus-tollens* [42] and [30].
   - the nonmonotonic properties of conditional probabilistic logic [4, 23, 24]

Here is an illustration of the kind of specific questions that you will takle in this project.

Take $\mathcal{SL}$ be the set of sentences built recursively from a finite propositional language $\mathcal{L}$ as usual, and let $P : \mathcal{SL} \to \{0, 1\}$ be a probability function, that is to say normalised w.r.t. classical tautologies and finitely additive with respect to classical disjunctions. Following [37] define, for each such probability function, for any $\theta, \varphi \in \mathcal{SL}$ and for any real number $t \in [0, 1]$ a *probabilistic consequence relation* $\mid\!\sim_{P,t}$ by letting

$$\theta \mid\!\sim_{P,t} \varphi \text{ if and only if } P(\theta \wedge \varphi) \geq tP(\theta).$$

In this context t is interpreted as a *threshold* past which one interprets $P(\varphi \mid \theta)$ as having *sufficiently high probability*. While originally developed to resolve negatively a conjecture of [25], threshold-based probabilistic consequence relations provides a very promising framework for casting a variety of key questions related to this project, including the so-called "Adam's thesis" [2] and the stability theory of belief [31].

1.3. **Profile.** The ideal prerequisites for this project are:

- a degree in Logic (or closely related fields)
- a confident command of classical logic
- working knowledge (e.g. as obtained in an undergraduate course) of probability or statistics.

Additional and very valuable assets include:

- knowledge of the history of modern logic
- working knowledge of abstract algebra
- graduate exposure to nonmonotonic logics

2. Probabilistic Proofs-as-Programs

2.1. **Background.** The computational interpretation of logical proofs, known as the Curry-Howard isomorphism [9, 27], has influenced the early tradition of logical systems for the verification of specification requirements satisfaction, e.g in [12, 15, 26], see [39, ch.7] for a historical and formal overview.

With the development of probabilistic interpretations of logics, and especially with the strengthening of their relevance due to the advent of Machine Learning techniques [16, 34, 36], the role of types and proofs in a probabilistic setting has been recently extensively explored in the literature, with the introduction of some forms of probability in calculi with types or natural deduction systems. Notice that this is a different research program than the probabilistic proof-checking of deterministic computations, see e.g. [18].

[38] introduces a λ-calculus augmented with special "probabilistic choice" constructs, i.e. terms of the form $M = \{p_1 M_1, \dots, p_n M_n\}$, meaning that term M has probability $p_1, \dots, p_n$ of reducing to one of the terms $M_1, \dots, M_n$ respectively. Unlike TPTND, [38] deals with judgements that do not have a context and uses a subtyping relation for the term reduction.

[6] introduces "probabilistic sequents" of the form $\Gamma \vdash_a^b \Delta$ that are interpreted as empirical statements of the form "the probability of $\Gamma \vdash \Delta$ lies in the interval $[a, b]$". Differently from this work, TPTND does not express explicitly probabilistic intervals, but only sharp probability values, while at the same time expressing such probability on output types, rather than on the derivability relation.

Finally, [17] introduces the logic $\mathrm{P}\Lambda_{\rightarrow}$ where it is possible to mix "basic" and "probabilistic" formulas, the former being similar to standard Boolean formulas, and the latter being formed starting from the concept of a "probabilsitic operator" $P_{\geq s} M : \sigma$ stating that the probability of $M : \sigma$ is equal to or greater than s. The semantics for $\mathrm{P}\Lambda_{\rightarrow}$ is Kripke-like and its axiomatisation is infinitary.

Most significantly for type-theoretical models of probabilistic reasoning, [3] introduces a quantitative logic with fuzzy predicates and conditioning of states. The computation rules of the system can be used for calculating conditional probabilities in two well-known examples of Bayesian reasoning in (graphical) models.

In the same family, [43] offers a Probabilistic Dependent Type Systems (PDTS) via a functional language based on a subsystem of intuitionistic type theory including dependent sums and products, expanded to include stochastic functions. We provide a sampling-based semantics for the language based on non-deterministic beta reduction. A probabilistic logic from the PDTS introduced as a direct result of the Curry-Howard isomorphism is derived, shown to provide a universal representation for finite discrete distributions.

In recent work [11], we have formulated the probabilistic typed natural deduction calculus TPTND, which offers an integrated way to deal with probability intervals (similarly to [17]) and probabilistic choices (similarly to [38]). However, unlike these languages, TPTND was mainly designed to reason about and derive trustworthiness properties of computational processes. In fact, differently from all these other systems, TPTND has an explicit syntax both to deal with the number of experiments, and to prove trust in a process whenever the empirically verified probability is close enough to the theoretical one. The application of TPTND is specifically targeted to the evaluation of trustworthiness of probabilistic computational processes like those underlying current AI applications.

2.2. **Aim of the Project.** The project has two main aims:

- a thorough review of the relevant literature on the proof- and type-theoretical approaches to probabilistic reasoning which can be brought under the label of *probabilistic Curry-Howard isomorphism*, including and extending the formal attempts listed in the Background section of this document;
- an orginal contribution extending the system TPTND [11] in one or more of the following non-exclusive directions

- the development of a verification protocol for probabilistic trustworthy computations, e.g. by extending the existing Coq protocol for trust presented in [40] with one of the available libraries for probabilistic reasoning, e.g. https://github.com/jtassarotti/polaris;
- the extension of TPTND with probabilistic intervals and imprecise probabilities;
- the use of TPTND in the verification of biased computations;
- taking into account a finite number of resources, especially for experiments
- the development of sound relational and state transition semantics.

2.3. **Profile.** The ideal prerequisites for this project are:

- a degree in Logic (or closely related fields)
- a confident command of intuitionistic logic
- working knowledge (e.g. as obtained in an undergraduate course) of probability or statistics.

Additional and very valuable assets include:

- knowledge of the verification debate and its sub-fields
- working knowledge of proof-checking techniques, e.g. Coq or HOL
- working knowledge of computational trust literature

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